Converse, Inverse, and Contrapositive Statements

This activity should be done after you have discussed conditional statements.

- Give students page to read the definitions of converse, inverse, and contrapositive statements. (It is on page 2).
- Write a conditional statement on the board and under it, write the converse, inverse, and contrapositive statement. Do not label the statements. (An example is given on page 3)
- Have the students write down one of the three statements on a sheet of paper or an index card.
- Assign one corner of the room to be the converse corner, one to be the inverse corner, and one to be the contrapositive corner. A sign hanging in each corner would be helpful.
- Have the students take their statement to the corner they believe they should go to.
- Have student discuss and debate if each person is in the correct corner, and if not, have them move to the appropriate corner. Let this continue until the students feel comfortable that they are in the correct place. By having students choose their own statements, they can't determine where they need to go just to have an equal number in each corner.
- Have them go back to their seats and put them in groups to do the sorting activity (It is on page 4)
- After the groups have finished, have them check their grouping with other groups to determine if they are in agreement. This is when you can decide how much input you need to give for your class and decide if you want to give the correct groups.
- If you have time, give the assignment as a round-table activity where each student does one problem, then they rotate papers, check each other's work, then do the next problem. If you don't have time, send it with them as homework. I often start the assignment as a roundtable, then have them complete the assignment at home. Then I know that every students has some correct work on their paper to take with them. (Assignment on page 5)

Reading: Converse, Inverse, and Contrapositive

Inverse, Converse, and Contrapositives Inverses: Generally the conditional if p then q is the connective most often used in reasoning. However; with some changes in words in the original statement, additional conditionals can be formed. These new conditionals are called the **inverse**, the **converse**, and the **contrapositive**.

Inverse is a statement formed by negating the hypothesis and conclusion of the original conditional. Symbolically, the inverse is written as ($\sim p \Rightarrow \sim q$). The symbols for the inverse may be read as: not *p*, *implies not q* OR if not *p*, then not *q*. E.g. Right angle is defined as- an angle whose measure is 90 degrees. If you are to write it as inverse statement, it can be done like: If an angle is not a right angle, then it does not measure 90.

Converse is a statement formed by interchanging the hypothesis and the conclusion i.e. original conditional ($p \Rightarrow q$) is written as ($q \Rightarrow p$). Notice that the symbols for converse may be read as 'q implies p' or 'if q, then p'. E.g. If you are to write the converse of: "If two lines don't intersect, then they are parallel", it can be written as "If two lines are parallel, then they don't intersect." It may be noted that the converse of a definition, must always be true. If this is not the case, then the definition is not valid. The converse is; therefore, can be taken as a helping tool in determining the validity of a definition.
Remember: a conditional ($p \Rightarrow q$) and its converse ($q \Rightarrow p$) may or may not be true. It is important that the truth value of each converse is judged on its own merits.

Contrapositive is a statement formed by negating both the hypothesis and conclusion ($p \Rightarrow q$) and also then interchanging these negations ($\sim q \Rightarrow \sim p$). The symbols for contrapositive may be read as 'not q implies not p' or 'if not q, then not p'. The contrapositive of a conditional statement always has the same truth value as the original statement. Therefore, the contrapositive of a definition is always true. E.g. the statement 'A triangle is a three-sided polygon' is true; its contrapositive, 'A polygon with greater or less than three sides is not a triangle' is true too.
Remember: a conditional ($p \Rightarrow q$) and its contrapositive ($\sim q \Rightarrow \sim p$) must have the same truth value. When a conditional is true, it's contrapositive is also true and when a conditional is false, it's contrapositive is also false.

Sample statements for the board.

Conditional Statement:

If it rains, then the game will be cancelled.

If it does not rain, then the game will not be cancelled.

If the game is not cancelled, then it has not rained.

If the game is cancelled, then it has rained.

Sorting Activity

In each box, you are given two statements. The first is a conditional statement, and the second could be its converse, inverse, or contrapositive. Cut the boxes apart and sort them into a converse group, an inverse group, and a contrapositive group.

If two angles are congruent, then they have the same measure. If two angles have the same measure, then they are congruent.
If it snows, then school will be cancelled. If it does not snow, then school will not be cancelled.
If you add two even numbers, then the sum will also be even. If the sum of two numbers is not even, then the two numbers you added were not even.
If a polygon has six sides, then it is a hexagon. If a polygon is not a hexagon, then it does not have six sides.
If a student has his driver's license, then he is at least 16 years old. If a student is at least 16 years old, then he has his driver's license.
If the sum of two angle measures is 90°, then they are complementary. If the sum of two angle measures is not 90°, then they are not complementary.
If an animal has hair, then it is a mammal. If an animal does not have hair, then it is not a mammal.
If the sum of the measures of the interior angles of a polygon is 180°, then the polygon is a triangle. If a polygon is not a triangle, then the sum of the measures of its interior angles is not 180°.
If you brush your teeth, then you will not get cavities. If you do not get cavities, then you brush your teeth.
If you lift weights, then you are strong. If you are strong, then you lift weights.
If a fruit is yellow, then it is a banana. If a fruit is not yellow, then it is not a banana.
If you smoke cigarettes, your breath will stink. If your breath does not stink, then you do not smoke cigarettes.

Name _____ Date_____

Conditional, Converse, Inverse, and Contrapositive Homework

Find the inverse, converse, and contrapositive of the statement. (You may need to rewrite the statement in if-then form first).

1. If you visited Dallas, then you've been to Texas.

   Converse: _____

   Inverse: _____

   Contrapositive: _____

2. If freezing rain is falling, then the air temperature is 32°F or less.

   Converse: _____

   Inverse: _____

   Contrapositive: _____

3. If $m\angle P = 90°$, then $\angle P$ is a right angle.

   Converse: _____

   Inverse: _____

   Contrapositive: _____

4. If two planes intersect, then their intersection is a line.

   Converse: _____

   Inverse: _____

   Contrapositive: _____

5. Using p and q below, write the symbolic statement in words. Assume that p and q are true.
   p: the value of x is 4 *q: 3x + 2 = 14*

   $p \rightarrow q$ _____

   $\sim p \rightarrow \sim q$ _____

   $q \rightarrow p$ _____

   $\sim q \rightarrow \sim p$ _____

6. *Writing Activity:* Write a conditional statement of your own where the original conditional, the converse, the inverse, and the contrapositive are all true. You may use the back to test out the truth values, if you would like.